

CHAPTER 11

Analysis of Variance: Factorial Design

Summary _____

A *factorial design* has one dependent variable and two or more independent variables called factors. Each level of one factor is paired with every level of the other factor. A factorial ANOVA tests three null hypotheses: one for each of the two independent variables and one for the interaction between the two independent variables.

Table 11.1 shows a 2×3 factorial design. Each participant receives one level of one independent variable and one level of the other independent variable. The participants in the upper left-hand cell of Table 11.1 get Level 1 of Factor A and Level 1 of Factor B. Their dependent-variable scores are entered into cell A_1B_1 .

		Independent Variable A			Means
		Level 1	Level 2	Level 3	
Independent Variable B	Level 1	Participant's scores	Participant's scores	Participant's scores	\bar{X}_{B_1}
	Level 2	Participant's scores	Participant's scores	Participant's scores	\bar{X}_{B_2}
	Means	\bar{X}_{A_1}	\bar{X}_{A_2}	\bar{X}_{A_3}	

Table 11.1 Illustration of a 2×3 Factorial Design

Table 11.1 also shows marginal means for Factor A and marginal means for Factor B. The means for Factor A (and) are used in *one* of the statistical tests that a factorial ANOVA produces, the main effect of Factor A. The marginal means for Factor B (and) are involved in a *second* statistical test, the main effect of Factor B.

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Each main effect is comparable to a one-way ANOVA, and may be interpreted in the same way as a one-way ANOVA, unless the interaction is significant.

The *third test* in a factorial ANOVA is for an interaction between the two independent variables. Interactions occur when the effect of changing levels of one independent variable depends upon which level of the other independent variable you are administering. That is, one independent variable influences the effect that a second independent variable has on the dependent variable.

Perhaps an example of a significant interaction will help. The effect of chilling a rat until its body temperature is 20°C (37°C is normal) is disastrous; the rat dies. Likewise, asphyxiating a rat causes death. What chance would you predict for a rat undergoing asphyxiation while its body temperature is lowered to 20°C? The usual prediction is (death)². For these circumstances, however, there is an interaction; the rat lives for a very long time under these conditions. Each condition separately would kill the animal, but together they do not kill. Table 11.2 presents these results using the format of a 2 × 2 factorial design.

		Body Temperature	
		37°C	20°C
Asphyxiation	No	Alive	Dead
	Yes	Dead	Alive

Table 11.2 Illustration of a Significant Interaction

The effect of changing from 37°C to 20 °C depends on whether or not the animal is being asphyxiated. If the animal is not being asphyxiated, changing from 37°C to 20 °C kills it. If the animal is being asphyxiated, the effect is just the opposite; changing from 37°C to 20 °C restores life. To sum up, there is significant interaction in Table 11.2; the effect of asphyxiation depends on body temperature.

Graphs of the cell means are helpful when interpreting the results of a factorial ANOVA. Line graph curves that are not parallel *may* indicate that an interaction has occurred. Likewise, bar graphs with stair steps that change size or direction *may* indicate that two independent variables interact.

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To make the three statistical tests that a factorial ANOVA allows, the total variability of all the scores is partitioned into four components. One component is the variability that is due to one of the factors. A second component is the variability due to the second factor. A third component is the variability due to the interaction. The final component is the error variability, which serves as an estimate of the population variability. Each of these components, divided by its degrees of freedom results in a mean square. Finally, the error mean square is divided into the mean square for one factor (a main effect), the mean square for the second factor (a second main effect), and into the mean square for interaction (the interaction effect). These divisions produce F values, which are interpreted using a table of the sampling distribution of F .

Tukey HSD tests may be used to make pairwise comparisons among the levels of a factor. These tests can be conducted for either or for both of the factors in a factorial ANOVA. In Table 11.1, a Tukey HSD test on Factor B is unnecessary; the F test on that factor tells you whether or not the two margin means are significantly different. Unfortunately, Tukey HSD tests are not appropriate if the interaction is significant.

Factorial design data can be analyzed with a factorial ANOVA if the dependent variable scores meet certain qualifications. These *assumptions* include the three that must be met by all data analyzed by ANOVA. These assumptions are that the populations the samples are from be *normally distributed* and have *equal variances*, and that *random assignment* (or random sampling) be used. In addition to these assumptions, the methods described in Chapter 11 require that the N 's in each cell be equal, that the scores be independent (not paired or related), and that the levels of the independent variable be chosen by the experimenter rather than at random.

Multiple-Choice Questions

1. As used in analysis of variance, the term factor means
 - (1) independent variable;
 - (2) dependent variable;
 - (3) extraneous variable;
 - (4) none of the above.

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2. Designs that are equal in the number of factors are
 - (1) Independent-samples t test and factorial ANOVA;
 - (2) paired-samples t test and factorial ANOVA;
 - (3) one-way ANOVA and factorial ANOVA;
 - (4) none of the above.

3. A social psychologist was interested in the effect of propaganda on attitudes of males and females. He measured attitudes toward democracy for all participants, delivered the propaganda, and remeasured attitudes toward democracy. He wanted to analyze the data using a 2×2 factorial with gender as one independent variable and before-and-after attitudes as the other. The techniques presented in the text will not permit this because
 - (1) the scores are not independent;
 - (2) an independent variable was not defined ;
 - (3) the dependent variable was not defined;
 - (4) the levels of the independent variable were not chosen at random by the experimenter.

4. A cell in a factorial ANOVA, refers to
 - (1) one level of the independent variable;
 - (2) one level of the dependent variable;
 - (3) one level of one independent variable and one level of a second independent variable;
 - (4) all the participants in the experiment.

5. When the cell means of a factorial design are presented as a line graph, a nonsignificant interaction is indicated by
 - (1) parallel lines;
 - (2) crossed lines;
 - (3) values of F less than 1.00;
 - (4) any of the above.

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6. Suppose you had a 2×2 factorial ANOVA with the four cell means and you knew that N was 6 for each cell. With this information you could not calculate
- (1) SS_A ;
 - (2) SS_B ;
 - (3) SS_{AB} ;
 - (4) SS_{error} .
7. A 3×5 factorial ANOVA has _____ independent variables.
- (1) 1;
 - (2) 2;
 - (3) 3;
 - (4) 5.
8. In a 4×4 factorial design with five subjects per cell, the df for the interaction F would be
- (1) 3, 70 df ;
 - (2) 6, 64 df ;
 - (3) 9, 64 df ;
 - (4) none of the above.
9. The term *main effect* refers to a comparison of
- (1) means;
 - (2) interactions;
 - (3) both (1) and (2);
 - (4) neither (1) nor (2).
10. An interaction means that
- (1) the cell means all differed;
 - (2) the margin means were identical;
 - (3) there was a difference between two variables;
 - (4) none of the above accurately describe an interaction.

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11. Mr. Jefferson, a farmer-philosopher-president, has a vineyard. On his fields 5-10-5 fertilizer (5 percent nitrogen, 10 percent potash, 5 percent potassium) causes a 10 percent increase in yield. 10-20-10 fertilizer causes a 15 percent increase in yield. Suppose both fertilizers were applied by Mr. Jefferson (whose philosophy is, "If a little bit does a little good, a whole lot will do a lot of good"). Which outcome below would indicate there was no interaction?
- (1) 10 percent increase;
 - (2) 15 percent increase;
 - (3) either (1) or (2);
 - (4) 25 percent increase.
12. "The means of the populations from which the samples were drawn are identical." This is a statement
- (1) of the null hypothesis;
 - (2) of one of the assumptions required of data analyzed with a factorial ANOVA;
 - (3) about an interaction;
 - (4) none of the above
13. Which of the following is (are) true?
- (1) $df_A + df_B = df_{AB}$;
 - (2) $MS_A + MS_B = MS_{AB}$;
 - (3) both (1) and (2);
 - (4) neither (1) nor (2).
14. In common with one-way ANOVA, factorial ANOVA has which of the following assumptions:
- (1) the dependent variable is assumed to be normally distributed;
 - (2) the population variances of all the populations samples are equal;
 - (3) both (1) and (2);
 - (4) neither (1) nor (2).

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15. One difference between the one-way ANOVA and factorial ANOVA is that
- (1) factorial ANOVA has two or more dependent variables;
 - (2) factorial ANOVA has an independent variable with more than 2 levels;
 - (3) factorial ANOVA has two independent variables;
 - (4) factorial ANOVA requires fewer assumptions about the data.
16. Suppose a 5×6 factorial ANOVA with 7 scores per cell produced $MS_A = 25$, $MS_B = 30$, $MS_{AB} = 15$, and $MS_{error} = 10$ (where A is the 5 in the 5×6). With $\alpha = .01$, a Tukey HSD
- (1) is not appropriate in this instance;
 - (2) would be significant for a difference of 2.00 between two A means;
 - (3) would be significant for a difference of 2.40 between two A means;
 - (4) both (2) or (3).
17. Suppose a 2×4 factorial ANOVA with 5 scores per cell produced $MS_A = 200$, $MS_B = 285$, $MS_{AB} = 225$, and $MS_{error} = 100$ (where B is the 4 in the 2×4). With $\alpha = .01$, a Tukey HSD
- (1) is not appropriate in this instance;
 - (2) would be significant for a difference of 10 between two B means;
 - (3) would be significant for a difference of 15 between two B means;
 - (4) neither (2) nor (3).
18. Which of the following would be reason enough not to calculate a Tukey HSD?
- (1) a significant interaction;
 - (2) a significant main effect;
 - (3) either (1) or (2);
 - (4) neither (1) nor (2).
19. A follow-up Tukey HSD is not necessary for main effects when the F value is based on _____ degrees of freedom.
- (1) 1, 20;
 - (2) 2, 20;
 - (3) 4, 20;
 - (4) 4, 10.

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20. Consider a factorial ANOVA in which the dependent variable is reaction time scores. Three different drugs are tested on both females and males. The researchers concluded that the effect of a drug did not depend on whether a person taking it was a female or male. The factorial ANOVA would certainly show that

- (1) there is no main effect for drug;
- (2) there is no main effect for gender;
- (3) there is no interaction;
- (4) all of the above.

Interpretation _____

1. The numbers are cell means. For each figure, indicate whether the interaction and the main effects are not significant or probably significant.

X

	B ₁	B ₂	B ₃
A ₁	5	10	15
A ₂	5	10	15

AB Interaction _____
 A Main Effect _____
 B Main Effect _____

Y

	B ₁	B ₂	B ₃
A ₁	15	10	15
A ₂	5	10	5

AB Interaction _____
 A Main Effect _____
 B Main Effect _____

2. The numbers are cell means. For each figure, indicate whether the interaction and the main effects are not significant or probably significant.

X

	B ₁	B ₂
A ₁	5	10
A ₂	10	5

AB Interaction _____
 A Main Effect _____
 B Main Effect _____

Y

AB Interaction _____

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	B ₁	B ₂
A ₁	5	10
A ₂	5	10

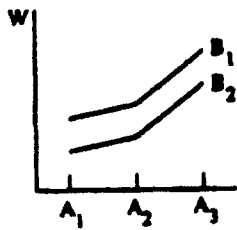
A Main Effect _____
 B Main Effect _____

Z

	B ₁	B ₂
A ₁	5	5
A ₂	5	10

AB Interaction _____
 A Main Effect _____
 B Main Effect _____

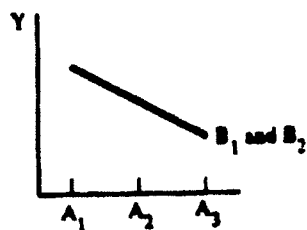
3. For each figure, indicate whether the interaction and the main effects are not significant or probably significant.



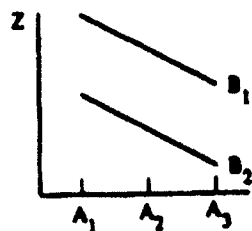
AB Interaction _____
 A Main effect _____
 B Main effect _____



AB Interaction _____
 A Main effect _____
 B Main effect _____



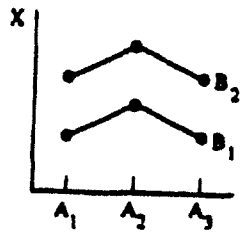
AB Interaction _____
 A Main effect _____
 B Main effect _____



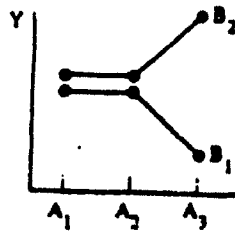
AB Interaction _____
 A Main effect _____
 B Main effect _____

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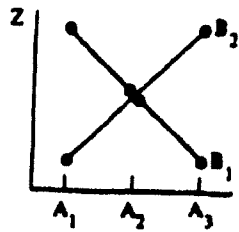
4. For each figure, indicate whether the interaction and the main effects are not significant or probably significant.



AB Interaction _____
 A Main effect _____
 B Main effect _____



AB Interaction _____
 A Main effect _____
 B Main effect _____



AB Interaction _____
 A Main effect _____
 B Main effect _____

5. Milton Rokeach thinks that racial and ethnic prejudice occurs, not because of differences in race and ethnicity, but because people assume that others differ in basic values. Studies similar to the following have been done to test Rokeach's hypothesis.

Participants were measured on prejudice and then separated into low scorers, moderate scorers, and high scorers. Next, participants read descriptions of people and for each person, they indicated the degree of intimacy they might expect to have with that person. Half the participants read descriptions of persons who differed from themselves in race but not in values; the other half read descriptions that differed in values but not in race. The design is a 2 x 3 factorial. Identify the independent and dependent variables. Examine the summary data that follow and write a description of what the data show.

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		Prejudice (A)		
		<u>Low (A₁)</u>	<u>Moderate</u> <u>(A₂)</u>	<u>High (A₃)</u>
Descriptions	Values(B ₁)	8.20	9.00	10.40
Differ (B)	Race (B ₂)	15.40	13.40	7.00

<u>Source</u>	<u>SS</u>	<u>df</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Prejudice (A)	54.07	2	27.03	7.51	< .01
Descriptions (B)	56.03	1	56.03	15.56	< .01
AB	150.87	2	75.43	20.95	< .01
Error	86.40	24	3.60		
Total	347.37	29			

6. A social psychologist recruited 30 male and female undergraduates to engage in a telephone conversation with a person of the opposite sex. Afterwards, the participants rated the person “as a person” on a 10-point scale. Each conversation in the experiment was with a confederate of the psychologist. Before the conversation, each participant was given a description of the other person, and embedded in the description was the second independent variable: physical attractiveness. The descriptions included one of three phrases, “truly gorgeous,” “kind of ordinary looking,” or “pretty ugly, really.” Examine the table of cell means and the ANOVA summary table. Write an explanation of what the data show.

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	Gender	
	<u>Males</u> <u>(A₁)</u>	<u>Females</u> <u>(A₂)</u>
Gorgeous (B ₁)	8.10	8.00
Ordinary (B ₂)	6.10	6.50
Ugly (B ₃)	4.60	4.70

<u>Source</u>	<u>SS</u>	<u>df</u>	<u>MS</u>	<u>F</u>	<u>p</u>
Gender (A)	.27	1	.27	0.09	> .05
Description (B)	115.63	2	57.82	18.50	< .01
AB	.63	2	.32	0.10	> .05
Error	168.80	54	3.13		
Total	285.33	59			

Problems

1. Carl Hovland (Hovland, Lumsdaine, & Sheffield, 1949) conducted a study during WW II to find out whether people were more likely to be persuaded by a message that gave just one side of an issue or by one that gave both sides. (In 1944, Germany was falling, and the attitude of many American soldiers was that the war with Japan would soon be over. Afraid that battle preparedness would fall off, the Army wanted to reverse this attitude. The question was “How should we present the argument?”) In one part of Hovland’s study, there were two independent variables—number of sides of the argument presented and educational level of the listeners. The dependent variable was the amount of attitude change produced. Below are summary data. Finish the analysis, and interpret the results.

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		High School Graduate	
		Yes (A_1)	No (A_2)
Argument	One-sided (B_1)	$\Sigma X_{A_1B_1} = 64$ $\Sigma X^2_{A_1B_1} = 808$ $N = 6$	$\Sigma X_{A_2B_1} = 123$ $\Sigma X^2_{A_2B_1} = 2603$ $N = 6$
	Two-sided (B_2)	$\Sigma X_{A_1B_2} = 64$ $\Sigma X^2_{A_1B_2} = 808$ $N = 6$	$\Sigma X_{A_2B_2} = 64$ $\Sigma X^2_{A_2B_2} = 808$ $N = 6$

2. Two approaches to learning a list of words are rote rehearsal and elaborative rehearsal. In rote rehearsal, a person repeats the word over and over. Some words such as truck and rose have high imagery and others such as honor and concept have low imagery. For elaborative rehearsal, a person gets a mental image or association to a word. The data below show the number of words correctly recalled from a list after using one of the rehearsal techniques. Perform a factorial ANOVA, compile a summary table, and write a conclusion.

High Imagery, Rote Strategy: 8, 6, 5, 6

High Imagery, Elaborative Strategy: 13, 10, 8, 15

Low Imagery, Rote Strategy: 6, 5, 3, 4

Low Imagery, Elaborative Strategy: 7, 5, 2, 3